# On the closest distance between a point and a convex body

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In this talk we fix a strictly convex body in the plane and a point in its exterior. We investigate the following problem with possible practical applications: find the point on the boundary of the fixed body, for which the distance to the given point is minimal. The focus of the paper is on the practical aspect of computational algorithm, which can be applied to obtain approximate or exact solution of the aforementioned problem.

Let $C$ be a plane closed strictly convex curve, and the origin of coordinate system lies in the region bounded by $C$. We denote by $p$ the support function of $C$ with respect to the origin. The support function $p$ is differentiable and the parametrization of $C$ in terms of this function is given by

$$z(t) = p(t)e^{it} + \dot{p}(t)ie^{it}, \tag{1}$$

cf. [1]. We assume that $z(0)$ lies in the first quadrant. First we find the equation

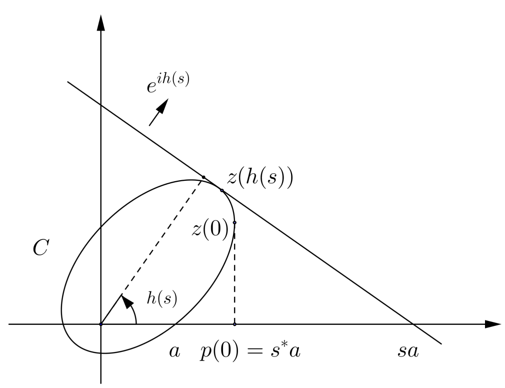


Figure 1: Definitions of $a$, $h(s)$, $sa$

of support line to $C$ passing through a given point $(b,0)$, where $b > p(0)$. We introduce the notations as on the Figure 1, where $h$ is a function of the variable

$s \in (s^*, +\infty)$ with values in the interval $\left(0, \frac{\pi}{2}\right)$. Let us introduce a function $f(u) = \frac{p(u)}{a\cos u}$. for $u \in \left(0, \frac{\pi}{2}\right)$. Then $f \circ h = \mathrm{id}$ and the function $f$ is invertible, so if $b = sa$ then our support line has the following equation

$$x + y\tan f^{-1}\left(\frac{b}{a}\right) - b = 0. \tag{2}$$

In the further part of the talk we assume that $C$ be a strictly convex curve given by (1) and $a = z(t^*) > 0$. If $C$ satisfies the condition $\operatorname{Im} z(0) < 0$, then the function $Q\colon (0, t^*) \to \mathbf{R}$ given by the formula $Q(u) = -\frac{\dot{p}(u)}{a\sin u}$ is positive-valued and strictly decreasing. We then prove the main theorem of the talk

**Theorem** *Let $C$ be a strictly convex curve given by* (1) *and $a = z(t^*) > 0$. If $b > a$ and* $\operatorname{Im} z(0) < 0$ *then the point $z\left(Q^{-1}\left(\frac{b}{a}\right)\right)$, realizes the shortest distance between $(b,0)$ and $C$.*

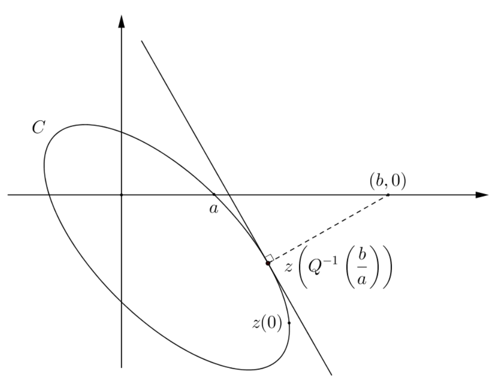


Figure 2: Point $z\left(Q^{-1}\left(\frac{b}{a}\right)\right)$ realizes the minimal distance

In general it is not trivial, or even impossible, to obtain the inverse of the function $Q$ but at the end of the talk we describe how to approximate its inverse, which gives us the possibility of finding the approximation yielding the shortest distance between a given point and a strictly convex curve. We introduce an algorithm, which applies the ideas presented above which can be divided into two parts, the first one is to be done for a given convex set, the second one for a given point. The algorithm will be illustrated on two examples.

# References


[1] Bonnesen, T.; Fenchel, W.; *Theorie der konvexen Körper*, Berlin-Heidelberg-New York: Springer-Verlag. (1974).